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13.1 Introduction to Space Frame Structures

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13.1 Introduction to Space Frame Structures

13.1.1 General Introduction

A growing interest in space frame structures has been witnessed worldwide over the last half century. The search for new structural forms to accommodate large unobstructed areas has always been the main objective of architects and engineers. With the advent of new building techniques and construction materials, space frames frequently provide the right answer and satisfy the requirements for lightness, economy, and speedy construction. Significant progress has been made in the process of the development of the space frame. A large amount of theoretical and experimental research programs was carried out by many universities and research institutions in various countries. As a result, a great deal of useful information has been disseminated and fruitful results have been put into practice.

In the past few decades, the proliferation of the space frame was mainly due to its great structural potential and visual beauty. New and imaginative applications of space frames are being demonstrated in the total range of building types, such as sports arenas, exhibition pavilions, assembly halls, transportation terminals, airplane hangars, workshops, and warehouses. They have been used not only on long-span roofs, but also on mid- and short-span enclosures as roofs, floors, exterior walls,
and canopies. Many interesting projects have been designed and constructed all over the world using a variety of configurations.

Some important factors that influence the rapid development of the space frame can be cited as follows. First, the search for large indoor space has always been the focus of human activities. Consequently, sports tournaments, cultural performances, mass assemblies, and exhibitions can be held under one roof. The modern production and the needs of greater operational efficiency also created demand for large space with a minimum interference from internal supports. The space frame provides the benefit that the interior space can be used in a variety of ways and thus is ideally suited for such requirements.

Space frames are highly statically indeterminate and their analysis leads to extremely tedious computation if by hand. The difficulty of the complicated analysis of such systems contributed to their limited use. The introduction of electronic computers has radically changed the whole approach to the analysis of space frames. By using computer programs, it is possible to analyze very complex space structures with great accuracy and less time involved.

Lastly, the space frame also has the problem of connecting a large number of members (sometimes up to 20) in space through different angles at a single point. The emergence of several connecting methods of proprietary systems has made great improvement in the construction of the space frame, which offered simple and efficient means for making connection of members. The exact tolerances required by these jointing systems can be achieved in the fabrication of the members and joints.

13.1.2 Definition of the Space Frame

If one looks at technical literature on structural engineering, one will find that the meaning of the space frame has been very diverse or even confusing. In a very broad sense, the definition of the space frame is literally a three-dimensional structure. However, in a more restricted sense, space frame means some type of special structure action in three dimensions. Sometimes structural engineers and architects seem to fail to convey with it what they really want to communicate. Thus, it is appropriate to define here the term space frame as understood throughout this section. It is best to quote a definition given by a Working Group on Spatial Steel Structures of the International Association [11].

A space frame is a structure system assembled of linear elements so arranged that forces are transferred in a three-dimensional manner. In some cases, the constituent element may be two-dimensional. Macroscopically a space frame often takes the form of a flat or curved surface.

It should be noted that virtually the same structure defined as a space frame here is referred to as latticed structures in a State-of-the-Art Report prepared by the ASCE Task Committee on Latticed Structures [2] which states:

A latticed structure is a structure system in the form of a network of elements (as opposed to a continuous surface). Rolled, extruded or fabricated sections comprise the member elements. Another characteristic of latticed structural system is that their load-carrying mechanism is three dimensional in nature.

The ASCE Report also specifies that the three-dimensional character includes flat surfaces with loading perpendicular to the plane as well as curved surfaces. The Report excludes structural systems such as common trusses or building frames, which can appropriately be divided into a series of planar frameworks with loading in the plane of the framework. In this section the terms space frames and latticed structures are considered synonymous.

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A space frame is usually arranged in an array of single, double, or multiple layers of intersecting members. Some authors define space frames only as double layer grids. A single layer space frame that has the form of a curved surface is termed as braced vault, braced dome, or latticed shell.

Occasionally the term space truss appears in the technical literature. According to the structural analysis approach, a space frame is analyzed by assuming rigid joints that cause internal torsions and moments in the members, whereas a space truss is assumed as hinged joints and therefore has no internal member moments. The choice between space frame and space truss action is mainly determined by the joint-connection detailing and the member geometry is no different for both. However, in engineering practice, there is no absolutely rigid or hinged joints. For example, a double layer flat surface space frame is usually analyzed as hinged connections, while a single layer curved surface space frame may be analyzed either as hinged or rigid connections. The term space frame will be used to refer to both space frames and space trusses.

13.1.3 Basic Concepts

The space frame can be formed either in a flat or a curved surface. The earliest form of space frame structures is a single layer grid. By adding intermediate grids and including rigid connecting to the joist and girder framing system, the single layer grid is formed. The major characteristic of grid construction is the omni-directional spreading of the load as opposed to the linear transfer of the load in an ordinary framing system. Since such load transfer is mainly by bending, for larger spans, the bending stiffness is increased most efficiently by going to a double layer system. The load transfer mechanism of curved surface space frame is essentially different from the grid system that is primarily membrane-like action. The concept of a space frame can be best explained by the following example.

**EXAMPLE 13.1:**

It is necessary to design a roof structure for a square building. Figure 13.1a and b show two different ways of roof framing. The roof system shown in Figure 13.1a is a complex roof comprised of planar latticed trusses. Each truss will resist the load acting on it independently and transfer the load to the columns on each end. To ensure the integrity of the roof system, usually purlins and bracings are used between trusses. In Figure 13.1b, latticed trusses are laid orthogonally to form a system of space latticed grids that will resist the roof load through its integrated action as a whole and transfer the loads to the columns along the perimeters. Since the loads can be taken by the members in three dimensions, the corresponding forces in space latticed grids are usually less than that in planar trusses, and hence the depth can be decreased in a space frame.

The same concept can be observed in the design of a circular dome. Again, there are two different ways of framing a dome. The dome shown in Figure 13.2a is a complex dome comprised of elements such as arches, primary and secondary beams, and purlins, which all lie in a plane. Each of these elements constitutes a system that is stable by itself. In contrast, the dome shown in Figure 13.2b is an assembly of a series of longitudinal, meridional, and diagonal members, which is a certain form of latticed shell. It is a system whose resisting capacity is ensured only through its integral action as a whole.

The difference between planar structures and space frames can be understood also by examining the sequence of flow of forces. In a planar system, the force due to the roof load is transferred successively through the secondary elements, the primary elements, and then finally the foundation. In each case, loads are transferred from the elements of a lighter class to the elements of a heavier class. As the sequence proceeds, the magnitude of the load to be transferred increases, as does the span of the element. Thus, elements in a planar structure are characterized by their distinctive ranks, not only judging by the size of their cross-sections, but also by the importance of the task assigned.
to them. In contrast, in a space frame system, there is no sequence of load transfer and all elements contribute to the task of resisting the roof load in accordance with the three-dimensional geometry of the structure. For this reason, the ranking of the constituent elements similar to planar structures is not observed in a space frame.

### 13.1.4 Advantages of Space Frames

1. One of the most important advantages of a space frame structure is its light weight. It is mainly due to the fact that material is distributed spatially in such a way that the load transfer mechanism is primarily axial—tension or compression. Consequently, all material in any given element is utilized to its full extent. Furthermore, most space frames are now constructed with steel or aluminum, which decreases considerably their self-weight. This is especially important in the case of long span roofs that led to a number of notable examples of applications.

2. The units of space frames are usually mass produced in the factory so that they can take full advantage of an industrialized system of construction. Space frames can be built from simple prefabricated units, which are often of standard size and shape. Such units can be easily transported and rapidly assembled on site by semi-skilled labor. Consequently, space frames can be built at a lower cost.

3. A space frame is usually sufficiently stiff in spite of its lightness. This is due to its three-dimensional character and to the full participation of its constituent elements. Engineers appreciate the inherent rigidity and great stiffness of space frames and their exceptional ability to resist unsymmetrical or heavy concentrated load. Possessing greater rigidity,
4. Space frames possess a versatility of shape and form and can utilize a standard module to generate various flat space grids, latticed shell, or even free-form shapes. Architects appreciate the visual beauty and the impressive simplicity of lines in space frames. A trend is very noticeable in which the structural members are left exposed as a part of the architectural expression. Desire for openness for both visual impact as well as the ability to accommodate variable space requirements always calls for space frames as the most favorable solution.

13.1.5 Preliminary Planning Guidelines

In the preliminary stage of planning a space frame to cover a specific building, a number of factors should be studied and evaluated before proceeding to structural analysis and design. These include not only structural adequacy and functional requirements, but also the aesthetic effect desired.

1. In its initial phase, structural design consists of choosing the general form of the building and the type of space frame appropriate to this form. Since a space frame is assembled from straight, linear elements connected at nodes, the geometrical arrangement of the elements—surface shape, number of layers, grid pattern, etc.—needs to be studied carefully in the light of various pertinent requirements.

2. The geometry of the space frame is an important factor to be planned which will influence both the bearing capacity and weight of the structure. The module size is developed from the overall building dimensions, while the depth of the grid (in case of a double layer), the size of cladding, and the position of supports will also have a pronounced effect upon it. For a curved surface, the geometry is also related to the curvature or, more specifically, to the rise of the span. A compromise between these various aspects usually has to be made to achieve a satisfactory solution.
3. In a space frame, connecting joints play an important role, both functional and aesthetic, which is derived from their rationality during construction and after completion. Since joints have a decisive effect on the strength and stiffness of the structure and compose around 20 to 30% of the total weight, joint design is critical to space frame economy and safety. There are a number of proprietary systems that are used for space frame structures. A system should be selected on the basis of quality, cost, and erection efficiency. In addition, custom-designed space frames have been developed, especially for long span roofs. Regardless of the type of space frame, the essence of any system is the jointing system.

4. At the preliminary stage of design, choosing the type of space frame has to be closely related to the constructional technology. The space frames do not have such sequential order of erection for planar structures and require special consideration on the method of construction. Usually a complete falsework has to be provided so that the structure can be assembled in the high place. Alternatively, the structure can be assembled on the ground, and certain techniques can be adopted to lift the whole structure, or its large part, to the final position.

13.2 Double Layer Grids

13.2.1 Types and Geometry

Double layer grids, or flat surface space frames, consist of two planar networks of members forming the top and bottom layers parallel to each other and interconnected by vertical and inclined web members. Double layer grids are characterized by the hinged joints with no moment or torsional resistance; therefore, all members can only resist tension or compression. Even in the case of connection by comparatively rigid joints, the influence of bending or torsional moment is insignificant.

Double layer grids are usually composed of basic elements such as:

- a planar latticed truss
- a pyramid with a square base that is essentially a part of an octahedron
- a pyramid with a triangular base (tetrahedron)

These basic elements used for various types of double-layer grids are shown in Figure 13.3.

![FIGURE 13.3: Basic elements of double layer grids.](image)
A large number of types of double layer grids can be formed by these basic elements. They are developed by varying the direction of the top and bottom layers with respect to each other and also by the positioning of the top layer nodal points with respect to the bottom layer nodal points. Additional variations can be introduced by changing the size of the top layer grid with respect to the bottom layer grid. Thus, internal openings can be formed by omitting every second element in a normal configuration. According to the form of basic elements, double layer grids can be divided into two groups, i.e., latticed grids and space grids. The latticed grids consist of intersecting vertical latticed trusses and form a regular grid. Two parallel grids are similar in design, with one layer directly over the top of another. Both top and bottom grids are directionally the same. The space grids consist of a combination of square or triangular pyramids. This group covers the so-called offset grids, which consist of parallel grids having an identical layout with one grid offset from the other in plane but remaining directionally the same, as well as the so-called differential grids in which two parallel top and bottom grids are of a different layout but are chosen to coordinate and form a regular pattern [20].

The type of double layer grid can be chosen from the following most commonly used framing systems that are shown in Figure 13.4a through j. In Figure 13.4, top chord members are depicted with heavy solid lines, bottom chords are depicted with light solid lines and web members with dashed lines, while the upper joints are depicted by hollow circles and bottom joints by solid circles. Different types of double layer grids are grouped and named according to their composition and the names in the parenthesis indicate those suggested by other authors.

Group 1. Composed of latticed trusses
1. Two-way orthogonal latticed grids (square on square) (Figure 13.4a). This type of latticed grid has the advantage of simplicity in configuration and joint detail. All chord members are of the same length and lie in two planes that intersect at 90° to each other. Because of its weak torsional strength, horizontal bracings are usually established along the perimeters.
2. Two-way diagonal latticed grids (Figure 13.4b). The layout of the latticed grids is exactly the same as Type 1 except it is offset by 45° from the edges. The latticed trusses have different spans along two directions at each intersecting joint. Since the depth is all the same, the stiffness of each latticed truss varies according to its span. The latticed trusses of shorter spans may be considered as a certain kind of support for latticed trusses of longer span, hence more spatial action is obtained.
3. Three-way latticed grids (Figure 13.4c). All chord members intersect at 60° to each other and form equilateral triangular grids. It is a stiff and efficient system that is adaptable to those odd shapes such as circular and hexagonal plans. The joint detail is complicated by numerous members intersecting at one point, with 13 members in an extreme case.
4. One-way latticed grids (Figure 13.4d). It is composed of a series of mutually inclined latticed trusses to form a folded shape. There are only chord members along the spanning direction; therefore, one-way action is predominant. Like Type 1, horizontal bracings are necessary along the perimeters to increase the integral stiffness.

Group 2A. Composed of square pyramids
5. Orthogonal square pyramid space grids (square on square offset) (Figure 13.4e). This is one of the most commonly used framing patterns with top layer square grids offset over bottom layer grids. In addition to the equal length of both top and bottom chord members, if the angle between the diagonal and chord members is 45°, then all members in the space grids will have the same length. The basic element is a square pyramid that is used in some proprietary systems as prefabricated units to form this type of space grid.
6. Orthogonal square pyramid space grids with openings (square on square offset with internal openings, square on larger square) (Figure 13.4f). The framing pattern is similar
to Type 5 except the inner square pyramids are removed alternatively to form larger grids in the bottom layer. Such modification will reduce the total number of members and consequently the weight. It is also visually affective as the extra openness of the space grids network produces an impressive architectural effect. Skylights can be used with this system.

7. Differential square pyramid space grids (square on diagonal) (Figure 13.4g). This is a typical example of differential grids. The two planes of the space grids are at 45° to each other which will increase the torsional stiffness effectively. The grids are arranged orthogonally in the top layer and diagonally in the bottom layer. It is one of the most efficient framing systems with shorter top chord members to resist compression and longer bottom chords to resist tension. Even with the removal of a large number of members, the system is still structurally stable and aesthetically pleasing.

8. Diagonal square pyramid space grids (diagonal square on square with internal openings, diagonal on square) (Figure 13.4h). This type of space grid is also of the differential layout, but with a reverse pattern from Type 7. It is composed with square pyramids connected at their apices with fewer members intersecting at the node. The joint detail is relatively simple because there are only six members connecting at the top chord joint and eight members at the bottom chord joint.

Group 2B. Composed of triangular pyramids

9. Triangular pyramid space grids (triangle on triangle offset) (Figure 13.4i). Triangular pyramids are used as basic elements and are connected at their apices, thus forming a pattern of top layer triangular grids offset over bottom layer grids. If the depth of the space grids is equal to \( \sqrt{2/3} \) chord length, then all members will have the same length.

10. Triangular pyramid space grids with openings (triangle on triangle offset with internal openings) (Figure 13.4j). Like Type 6, the inner triangular pyramids may also be removed alternatively. As the figure shown, triangular grids are formed in the top layer while triangular and hexagonal grids are formed in the bottom layer. The pattern in the bottom layer may be varied depending on the ways of removal. Such types of space grids have a good open feeling and the contrast of the patterns is effective.

### 13.2.2 Type Choosing

In the preliminary stage of design, it is most important to choose an appropriate type of double layer grid that will have direct influence on the overall cost and speed of construction. It should be determined comprehensively by considering the shape of the building plan, the size of the span, supporting conditions, magnitude of loading, roof construction, and architectural requirements. In general, the system should be chosen so that the space grid is built of relatively long tension members and short compression members.

In choosing the type, the steel weight is one of the important factors for comparison. If possible, the cost of the structure should also be taken into account, which is complicated by the different costs of joints and members. By comparing the steel consumption of various types of double layer grids with rectangular plans and supported along perimeters, it was found that the aspect ratio of the plan, defined here as the ratio of a longer span to a shorter span, has more influence than the span of the double layer grids. When the plan is square or nearly square (aspect ratio = 1 to 1.5), two-way latticed grids and all space grids of Group 2A, i.e., Type 1, 2, and 5 through 8, could be chosen. Of these types, the diagonal square pyramid space grids or differential square pyramid space grids have the minimum steel weight. When the plan is comparatively narrow (aspect ratio = 1.5 to 2), then those double layer grids with orthogonal gird systems in the top layer will consume less steel than
FIGURE 13.4: Framing system of double layer grids.
FIGURE 13.4: (Continued) Framing system of double layer grids.
those with a diagonal grid system. Therefore, two-way orthogonal latticed grids, orthogonal square pyramid space grids, and also those with openings and differential square pyramid space grids, i.e., Types 1, 5, 6, and 7, could be chosen. When the plan is long and narrow, the type of one-way latticed grid is the only selection. For square or rectangular double layer grids supported along perimeters on three sides and free on the other side, the selection of the appropriate types for different cases is essentially the same. The boundary along the free side should be strengthened either by increasing the depth or number of layers. Individual supporting structures such as trusses or girders along the free side are not necessary.

In case the double layer grids are supported on intermediate columns, type could be chosen from two-way orthogonal latticed grids, orthogonal square pyramid space grids, and also those with openings, i.e., Types 1, 5, and 6. If the supports for multi-span double layer grids are combined with those along perimeters, then two-way diagonal latticed grids and diagonal square pyramid space grids, i.e., Types 2 and 8, could also be used.

For double layer grids with circular, triangular, hexagonal, and other odd shapes supporting along perimeters, types with triangular grids in the top layer, i.e., Types 3, 9, and 10, are appropriate for use.

The recommended types of double layer grids are summarized in Table 13.1 according to the shape of the plan and their supporting conditions.

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### 13.2.3 Method of Support

Ideal double layer grids would be square, circular, or other polygonal shapes with overhanging and continuous supports along the perimeters. This will approach more of a plate type of design which minimizes the maximum bending moment. However, the configuration of the building has a great number of varieties and the support of the double layer grids can take the following locations:

1. **Support along perimeters**—This is the most commonly used support location. The supports of double layer grids may directly rest on the columns or on ring beams connecting the columns or exterior walls. Care should be taken that the module size of grids matches the column spacing.

2. **Multi-column supports**—For single-span buildings, such as a sports hall, double layer grids can be supported on four intermediate columns as shown in Figure 13.5a. For buildings such as workshops, usually multi-span columns in the form of grids as shown in Figure 13.5b are used. Sometimes the column grids are used in combination with supports along perimeters as shown in Figure 13.5c. Overhangs should be employed where possible in order to provide some amount of stress reversal to reduce the interior chord forces and deflections. For those double layer grids supported on intermediate columns, it is best to design with overhangs, which are taken as 1/4 to 1/3 of the mid-span. Corner supports should be avoided if possible because they cause large forces in the edge chords. If only four supports are to be provided, then it is more desirable to locate them in the middle of the sides rather than at the corners of the building.

3. **Support along perimeters on three sides and free on the other side**—For buildings of a rectangular shape, it is necessary to have one side open, such as in the case of an airplane hanger or for future extension. Instead of establishing the supporting girder or truss on the free side, triple layer grids can be formed by simply adding another layer of several module widths (Figure 13.6). For shorter spans, it can also be solved by increasing the depth of the double layer grids. The sectional area of the members along the free side will increase accordingly.

The columns for double layer grids must support gravity loads and possible lateral forces. Typical types of support on multi-columns are shown in Figure 13.7. Usually the member forces around the support will be excessively large, and some means of transferring the loads to columns are necessary. It may carry the space grids down to the column top by an inverted pyramid as shown in Figure 13.7a or by triple layer grids as shown in Figure 13.7b, which can be employed to carry skylights. If necessary, the inverted pyramids may be extended down to the ground level as shown in Figure 13.7c. The spreading out of the concentrated column reaction on the space grids reduces the maximum chord and web member forces adjacent to the column supports and reduces the effective spans. The use of a vertical strut on column tops as shown in Figure 13.7d enables the space grids to be supported on top chords, but the vertical strut and the connecting joint have to be very strong. The use of
crosshead beams on column tops as shown in Figure 13.7e produces the same effect as the inverted pyramid, but usually costs more in material and special fabrication.
13.2.4 Design Parameters

Before any work can proceed on the analysis of a double layer grid, it is necessary to determine the depth and the module size. The depth is the distance between the top and bottom layers and the module is the distance between two joints in the layer of the grid (see Figure 13.8). Although these two parameters seem simple enough to determine, they will play an important role on the economy of the roof design. There are many factors influencing these parameters, such as the type of double layer grid, the span between the supports, the roof cladding, and also the proprietary system used. In fact, the depth and module size are mutually dependent which is related by the permissible angle between the center line of web members and the plane of the top and bottom chord members. This should be less than 30° or the forces in the web members and the length will be relatively excessive, but not greater than 60° or the density of the web members in the grid will become too high. For some of the proprietary systems, the depth and/or module are all standardized.

The depth and module size of double layer grids are usually determined by practical experience. In some of the paper and handbooks, figures on these parameters are recommended and one may find the difference is quite large. For example, the span-depth ratio varies from 12.5 to 25, or even more. It is usually considered that the depth of the space frame can be relatively small when compared with more conventional structures. This is generally true because double layer grids produce smaller deflections under load. However, depths that are small in relation to span will tend to use smaller modules and hence a heavier structure will result. In the design, almost unlimited possibilities exist in practice for the choice of geometry. It is best to determine these parameters through structural optimization.

Works have been done on the optimum design of double layer grids supported along perimeters. In an investigation by Lan [14], seven types of double layer grids were studied. The module dimension and depth of the space frame are chosen as the design variables. The total cost is taken as the objective function which includes the cost of members and joints as well as the roofing systems and enclosing walls. Such assumption makes the results realistic to a practical design. A series of double layer grids of different types spanning from 24 to 72 m was analyzed by optimization. It was found that the optimum design parameters were different for different types of roof systems. The module number generally increases with the span, and the steel purlin roofing system allows larger module sizes than that of reinforced concrete. The optimum depth is less dependent on the span and smaller depth can be used for a steel purlin roofing system. It should be observed that a smaller member density will lead to a grid with relatively few nodal points and thus the least possible production costs for nodes, erection expense, etc.

Through regression analysis of the calculated values by optimization method where the costs are within 3% optimum, the following empirical formulas for optimum span-depth ratios are obtained. It was found that the optimum depths are distributed in a belt and all the span-depth ratios within such range will give optimum effect in construction.
For a roofing system composed of reinforced concrete slabs

\[ L/d = 12 + 2 \]  

(13.1)

For a roofing system composed of steel purlins and metal decks

\[ L/d = (510 - L)/34 + 2 \]  

(13.2)

where \( L \) is the short span and \( d \) is the depth of the double layer grids.

Few data could be obtained from the past works. Regarding the optimum depth for steel purlin roofing systems, Geiger suggested the span-depth ratio to be varied from 10 to 20 with less than 10% variation in cost. Motro recommended a span-depth ratio of 15. Curves for diagonal square pyramid space grids (diagonal on square) were given by Hirata et al. and an optimum ratio of 10 was suggested. In the earlier edition of the Specifications for the Design and Construction of Space Trusses issued in China, the span-depth ratio is specified according to the span. These figures were obtained through the analysis of the parameters used in numerous design projects. A design handbook for double layer grids also gives graphs for determining upper and lower bounds of module dimension and depth. The relation between depth and span obtained from Equation 13.2 and relevant source is shown in Figure 13.9. For short and medium spans, the optimum values are in good agreement with those obtained from experience. It is noticeable that the span-depth ratio should decrease with the span, yet an increasing tendency is found from experience which gives irrationally large values for long spans.

\[ \text{FIGURE 13.9: Relation between depth and span of double layer grids.} \]

In the revised edition of the Specification for the Design and Construction of Space Trusses issued in China, appropriate values of module size and depth for commonly used double layer grids simply supported along the perimeters are given. Table 13.2 shows the range of module numbers of the top chord and the span-depth ratios prescribed by the Specifications.
<table>
<thead>
<tr>
<th>Type of double layer grids</th>
<th>Module number</th>
<th>Span-depth ratio</th>
<th>Module number</th>
<th>Span-depth ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 5, 6</td>
<td>(2 – 4) + 0.2L</td>
<td>2</td>
<td>(6 – 8) + 0.08L</td>
<td>10 – 14</td>
</tr>
<tr>
<td>2, 7, 8</td>
<td>(6 – 8) + 0.08L</td>
<td></td>
<td>(13 – 17) – 0.03L</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1. L Denotes the shorter span in meters. 2. When the span is less than 18 m, the number of the module may be decreased.

### 13.2.5 Cambering and Slope

Most double layer grids are sufficiently stiff, so cambering is often not required. Cambering is considered when the structure under load appears to be sagging and the deflection might be visually undesirable. It is suggested that the cambering be limited to 1/300 of the shorter span. As shown in Figure 13.10, cambering is usually done in (a) cylindrical, (b) ridge or (c, d) spherical shape. If the grid is being fabricated on site by welding, then almost any type of camber can be obtained as this is just a matter of setting the joint nodes at the appropriate levels. If the grid components are fabricated in the factory, then it is necessary to standardize the length of the members. This can be done by keeping either the top or bottom layer chords at the standard length, and altering the other either by adding a small amount to the length of each member or subtracting a small amount from it to generate the camber required.

![Figure 13.10: Ways of cambering.](image)

Sometimes cambering is suggested so as to ensure that the rainwater drains off the roof quickly to avoid ponding. This does not seem to be effective especially when cambering is limited. To solve the water run-off problem in those locations with heavy rains, it is best to form a roof slope by the following methods (Figure 13.11):

1. Establishing short posts of different height on the joints of top layer grids.
2. Varying the depth of grids.
3. Forming a slope for the whole grid.
4. Varying the height of supporting columns.

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13.2.6 Methods of Erection

The method chosen for erection of a space frame depends on its behavior of load transmission and constructional details, so that it will meet the overall requirements of quality, safety, speed of construction, and economy. The scale of the structure being built, the method of jointing the individual elements, and the strength and rigidity of the space frame until its form is closed must all be considered. The general methods of erecting double layer grids are as follows. Most of them can also be applied to the construction of latticed shells.

1. Assembly of space frame elements in the air—Members and joints or prefabricated sub-assembly elements are assembled directly on their final position. Full scaffoldings are usually required for such types of erection. Sometimes only partial scaffoldings are used if cantilever erection of a space frame can be executed. The elements are fabricated at the shop and transported to the construction site and no heavy lifting equipment is required. It is suitable for all types of space frame with bolted connections.

2. Erection of space frames by strips or blocks—The space frame is divided on its plane into individual strips or blocks. These units are fabricated on the ground level, then hoisted up into the final position and assembled on the temporary supports. With more work being done on the ground, the amount of assembling work at high elevation is reduced. This method is suitable for those double layer grids where the stiffness and load-resisting behavior will not change considerably after dividing into strips or blocks, such as two-way orthogonal latticed grids, orthogonal square pyramid space grids, and the those with openings. The size of each unit will depend on the hoisting capacity available.

3. Assembly of space frames by sliding element in the air—Separate strips of space frame are assembled on the roof level by sliding along the rails established on each side of the building. The sliding units may either slide one after another to the final position and then assembled together or assembled successively during the process of sliding. Thus, the erection of a space frame can be carried out simultaneously with the construction work underneath, which leads to savings of construction time and cost of scaffoldings. The sliding technique is relatively simple, requiring no special lifting equipment. It is suitable for orthogonal grid systems where each sliding unit will remain geometrically non-deferrable.

4. Hoisting of whole space frames by derrick masts or cranes—The whole space frame is assembled on the ground level so that most of the assembling work can be done before hoisting. This will result in an increased efficiency and better quality. For short and medium spans, the space frame can be hoisted up by several cranes. For long-span space frames, derrick masts are used as the support and electric winches as the lifting power. The whole space frame can be translated or rotated in the air and then seated on its final position. This method can be employed to all types of double layer grids.

5. Lifting-up the whole space frame—This method also has the benefit or assembling space frames on the ground level, but the structure cannot move horizontally during lifting.
Conventional equipment used is hydraulic jacks or lifting machines for lift-slab construction. An innovative method has been developed by using the center hole hydraulic jacks for slipforming. The space frame is lifted up simultaneously with the slipforms for r.c. columns or walls. This lifting method is suitable for double layer grids supported along perimeters or on multi-point supports.

6. Jacking-up the whole space frame—Heavy hydraulic jacks are established on the position of columns that are used as supports for jacking-up. Occasionally roof claddings, ceilings, and mechanical installations are also completed with the space frame on the ground level. It is appropriate for use in space frames with multi-point supports, the number of which is usually limited.

13.3 Latticed Shells

13.3.1 Form and Layer

The main difference between double layer grids and latticed shells is the form. For a double layer grid, it is simply a flat surface. For latticed shell, the variety of forms is almost unlimited. A common approach to the design of latticed shells is to start with the consideration of the form—a surface curved in space. The geometry of basic surfaces can be identified, according to the method of generation, as the surface of translation and the surface of rotation. A number of variations of form can be obtained by taking segments of the basic surfaces or by combining or adding them. In general, the geometry of surface has a decisive influence on essentially all characteristics of the structure: the manner in which it transfers loads, its strength and stiffness, the economy of construction, and finally the aesthetic quality of the completed project.

Latticed shells can be divided into three distinct groups forming singly curved, synclastic, and anticlastic surfaces. A barrel vault (cylindrical shell) represents a typical developable surface, having a zero curvature in the direction of generatrices. A spherical or elliptical dome (spheroid or elliptic paraboloid) is a typical example of a synclastic shell. A hyperbolic paraboloid is a typical example of an anticlastic shell.

Besides the mathematical generation of surface systems, there are other methods for finding shapes of latticed shells. Mathematically the surface can be defined by a high degree polynomial with the unknown coefficients determined from the known shape of the boundary and the known position of certain points at the interior required by the functional and architectural properties of the space. Experimentally the shape can be obtained by loading a net of chain wires, a rubber membrane, or a soap membrane in the desired manner. In each case the membrane is supported along a predetermined contour and at predetermined points. The resulting shape will produce a minimal surface that is characterized by a least surface area for a given boundary and also constant skin stress. Such experimental models help to develop an understanding about the nature of structural forms.

The inherent curvature in a latticed shell will give the structure greater stiffness. Hence, latticed shells can be built in single layer grids, which is a major difference from double layer grid. Of course, latticed shells may also be built in double layer grids. Although single layer and double layer latticed shells are similar in shape, the structural analysis and connecting detail are quite different. The single layer latticed shell is a structural system with rigid joints, while the double layer latticed shell has hinged joints. In practice, single layer latticed shells of short span with lightweight roofing may also be built with hinged joints. The members and connecting joints in a single layer shell of large span will resist not only axial forces as in a double layer shell, but also the internal moments and torsions. Since the single layer latticed shells are easily liable to buckling, the span should not be too large. There is no distinct limit between single and double layer, which will depend on the type of shell, the geometry and size of the framework, and the section of members.
13.3.2 Braced Barrel Vaults

The braced barrel vault is composed of member elements arranged on a cylindrical surface. The basic curve is a circular segment; however, occasionally a parabola, ellipse, or funicular line may also be used. Figure 13.12 shows the typical arrangement of a braced barrel vault. Its structural behavior depends mainly on the type and location of supports, which can be expressed as $L/R$, where $L$ is the distance between the supports in longitudinal direction and $R$ is the radius of curvature of the transverse curve.

If the distance between the supports is long and usually edge beams are used in the longitudinal direction (Figure 13.12a), the primary response will be beam action. For $1.67 < L/R < 5$, the barrel vaults are called long shells, which can be visualized as beams with curvilinear cross-sections. The beam theory with the assumption of linear stress distribution may be applied to barrel vaults that are of symmetrical cross-section and under uniform loading if $L/R > 3$. This class of barrel vault will have longitudinal compressive stresses near the crown of the vault, longitudinal tensile stresses towards the free edges, and shear stresses towards the supports.

As the distance between transverse supports becomes closer, or as the dimension of the longitudinal span becomes smaller than the dimension of the shell width such that $0.25 < L/R < 1.67$, then the primary response will be arch action in the transverse direction (Figure 13.12b). The barrel vaults are called short shells. Their structural behavior is rather complex and dependent on their geometrical proportions. The force distribution in the longitudinal direction is no longer linear, but in a curvilinear manner, trusses or arches are usually used as the transverse supports.

When a single braced barrel vault is supported continuously along its longitudinal edges on foundation blocks, or the ratio of $L/R$ becomes very small, i.e., $< 0.25$ (Figure 13.12c), the forces are carried directly in the transverse direction to the edge supports. Its behavior may be visualized as the response of parallel arches. Displacement in the radial direction is resisted by circumferential bending stiffness. Such type of barrel vault can be applied to buildings such as airplane hangars or gymnasiums where the wall and roof are combined together.

There are several possible types of bracing that have been used in the construction of single layer braced barrel vaults. Figure 13.13 shows five principle types:

1. Orthogonal grid with single bracing of Warren truss (a)
2. Orthogonal grid with single bracing of Pratt truss (b)
3. Orthogonal grid with double bracing (c)
The first three types of braced barrel vaults can be formed by composing latticed trusses with the difference in the arrangement of bracings (Figures 13.13a, b, and c). In fact, the original barrel vault was introduced by Foppl. It consists of several latticed trusses, spanning the length of the barrel and supported on the gables. After connection of the longitudinal booms of the latticed trusses, they became a part of the braced barrel vault of the single layer type.

The popular diamond-patterned lamella type of braced barrel vault consists of a number of interconnected modular units forming a rhombus shaped grid pattern (Figure 13.13d). Each unit, which is twice the length of the side of a diamond, is called a lamella. Lamella roofs proved ideal for prefabricated construction as all the units are of standard size. They were originally constructed of timber, but with the increase of span, steel soon became the most frequently used material.

To increase the stability of the structure and to reduce the deflections under unsymmetrical loads, purlins were employed for large span lamella barrel vaults. This created the three-way grid type of bracing and became very popular (Figure 13.13e). The three-way grid enables the construction of such systems using equilateral triangles composed of modular units, which are of identical length and can be connected with simple nodes.
Research investigations have been carried out on braced barrel vaults. One aspect of this research referred to the influence of different types of bracing on the resulting stress distribution. The experimental tests on the models proved that there are significant differences in the behavior of the structures, and the type of bracing has a fundamental influence upon the strength and load-carrying capacity of the braced barrel vaults. The three-way single layer barrel vaults exhibited a very uniform stress distribution under uniformly distributed load, and much smaller deflections in the case of unsymmetrical loading than for any of the other types of bracing. The experiments also showed that large span single layer braced barrel vaults are prone to instability, especially under the action of heavy unsymmetrical loads and that the rigidity of joints can exert an important influence on the overall stability of the structure.

For double layer braced barrel vaults, if two- or three-way latticed trusses are used to form the top and bottom layers of the latticed shell, the grid pattern is identical as shown in Figure 13.13 for single layer shells. If square or triangular pyramids are used, either the top or bottom layer grid may follow the same pattern as shown in Figure 13.13.

The usual height-to-width ratio for long shells varies from 1/5 to 1/7.5. When the barrel vault is supported along the longitudinal edges, then the height can be increased to 1/3 chord width. For long shells, if the longitudinal span is larger than 30 m, or for barrel vaults supported along longitudinal edges with a transverse span larger than 25 m, double layer grids are recommended. The thickness of the double layer barrel vault is usually taken from 1/20 to 1/40 of the chord width.

13.3.3 Braced Domes

Domes are one of the oldest and well-established structural forms and have been used in architecture since the earliest times. They are of special interest to engineers as they enclose a maximum amount of space with a minimum surface and have proved to be very economical in terms of consumption of constructional materials. The stresses in a dome are generally membrane and compressive in the most part of the shell except circumferential tensile stresses near the edge and small bending moments at the junction of the shell and the ring beam. Most domes are surfaces of revolution. The curves used to form the synclastic shell are spherical, parabolic, or elliptical covering circular or polygonal areas. Out of a large variety of possible types of braced domes, only four or five types proved to be frequently used in practice. They are shown in Figure 13.14.

1. Ribbed domes (a)
2. Schwedler domes (b)
3. Three-way grid domes (c)
4. Lamella domes (d, e)
5. Geodesic domes (f)

Ribbed domes are the earliest type of braced domes that were constructed (Figure 13.14a). A ribbed dome consists of a number of identical meridional solid girders or trusses, interconnected at the crown by a compression ring. The ribs are also connected by concentric rings to form grids in a trapezium shape. The ribbed dome is usually stiffened by a steel or reinforced concrete tension ring at its base.

A Schwedler dome also consists of meridional ribs connected together to a number of horizontal polygonal rings to stiffen the resulting structure so that it will be able to take unsymmetrical loads (Figure 13.14b). Each trapezium formed by intersecting meridional ribs with horizontal rings is subdivided into two triangles by a diagonal member. Sometimes the trapezium may also be subdivided by two cross-diagonal members. This type of dome was introduced by a German engineer, J.W. Schwedler, in 1863. The great popularity of Schwedler domes is due to the fact that, on the assumption of pin-connected joints, the structure can be analyzed as statically determinate. In practice, in addition
to axial forces, all the members are also under the action of bending and torsional moments. Many attempts have been made in the past to simplify their analysis, but precise methods of analysis using computers have finally been applied to find the actual stress distribution.

The construction of a three-way grid dome is self-explanatory. It may be imagined as a curved form of three-way double layer grids (Figure 13.14c). It can also be constructed in single layer for the dome. The Japanese "Diamond Dome" system by Tomoeumi Iron Works belongs to this category. The theoretical analysis of three-way grid domes shows that even under unsymmetrical loading the forces in this configuration are very evenly distributed leading to economy in material consumption.

A Lamella dome is formed by intersecting two-way ribs diagonally to form a rhombus-shaped grid pattern. As in a lamella braced barrel vault, each lamella element has a length that is twice the length of the side of a diamond. The lamella dome can be distinguished further from parallel and curved domes. For a parallel lamella as shown in Figure 13.14d, the circular plan is divided into several sectors (usually six or eight), and each sector is subdivided by parallel ribs into rhombus grids of the same size. This type of lamella dome is very popular in the U.S. It is sometimes called a Kiewitt dome, named after its developer. For a curved lamella as shown in Figure 13.14e, rhombus grids of different size, gradually increasing from the center of the dome, are formed by diagonal ribs along the radial lines. Sometimes, for the purpose of establishing purlins for roof decks, concentric rings are introduced and a triangular network is generated.

FIGURE 13.14: Braced domes.
The geodesic dome was developed by the American designer Buckminster Fuller, who turned architects’ attention to the advantages of braced domes in which the elements forming the framework of the structure are lying on the great circle of a sphere. This is where the name “geodesic” came from (Figure 13.14f). The framework of these intersecting elements forms a three-way grid comprising virtually equilateral spherical triangles. In Fuller’s original geodesic domes, he used an icosahedron as the basis for the geodesic subdivision of a sphere, then the spherical surface is divided into 20 equilateral triangles as shown in Figure 13.15a. This is the maximum number of equilateral triangles into which a sphere can be divided. For domes of larger span, each of these triangles can be subdivided into six triangles by drawing medians and bisecting the sides of each triangle. It is therefore possible to form 15 complete great circles regularly arranged on the surface of a sphere (see Figure 13.15b). Practice shows that the primary type of bracing, which is truly geodesic, is not sufficient because it would lead to an excessive length for members in a geodesic dome. Therefore, a secondary bracing has to be introduced. To obtain a more or less regular network of the bracing bars, the edges of the basic triangle are divided modularly. The number of modules into which each edge of the spherical icosahedron is divided depends mainly on the size of the dome, its span, and the type of roof cladding. This subdivision is usually referred to as “frequency” as depicted in Figure 13.15c. It must be pointed out that during such a subdivision, the resulting triangles are no longer equilateral. The members forming the skeleton of the dome show slight variation in their length. As the frequency of the subdivision increases, the member length reduces, and the number of components as well as the types of connecting joints increases. Consequently, this reflects in the increase of the final price of the geodesic dome, and is one of the reasons why geodesic domes, in spite of their undoubted advantages for smaller spans, do not compare equally well with other types of braced domes for larger spans.

The rise of a braced dome can be as flat as 1/6 of the diameter or as high as 3/4 of the diameter which will constitute a greater part of a sphere. For diameter of braced domes larger than 60 m, double layer grids are recommended. The ratio of the depth to the diameter is in the range of 1/30 to 1/50. For long spans, the depth can be taken as small as \( \frac{1}{100} \) of diameter.

The subdivision of the surface of a braced dome can also be considered by using one of the following three methods. The first method is based on the surface of revolution. The first set of lines of division is drawn as the meridional lines from the apex. Next, circumferential rings are added. This results in a ribbed dome and further a Schwedler dome. Alternately, the initial set may be taken as a series of spiral arcs, resulting in a division of the surface into triangular units as uniform as possible. This is achieved by drawing great circles in three directions as shown in the case of a grid dome. A noteworthy type of division of a braced dome is the parallel lamella dome which is obtained by combining the first and second methods described above. The third method of subdivision results from projecting the edges of in-polyhedra onto the spherical surface, and then inscribing a triangular network of random frequency into this basic grid. A geodesic dome represents an application of this method, with the basic field derived from the icosahedron further subdivided with equilateral triangles.

### 13.3.4 Hyperbolic Paraboloid Shells

The hyperbolic paraboloid or hypar is a translational surface formed by sliding a concave paraboloid, called a generatrix, parallel to itself along a convex parabola, called a directrix, which is perpendicular to the generatrix (Figure 13.16a). By cutting the surface vertically, parabolas can be obtained and by cutting horizontally hyperbolas can be obtained. Such surfaces can also be formed by sliding a straight line along two other straight lines skewed with respect to each other (Figure 13.16b). The hyperbolic paraboloid is a doubly ruled surface; it can be defined by two families of intersecting straight lines that form in plan projection a rhombic grid. This is one of the main advantages of a hyperbolic paraboloid shell. Although it has a double curvature anticlastic surface, it can be built by using linear structural members only. Thus, single layer hypar shells can be fabricated from straight beams and double layer hypar shells from linear latticed trusses. The single hypar unit shown in
Figure 13.15 is suitable for use in building of square, rectangular, or elliptic plan. In practice, there exist an infinite number of ways of combining hypar units to enclose a given building space.

A shallow hyperbolic paraboloid under uniform loading acts primarily as a shear system, where the shear forces, in turn, cause diagonal tension and compression. The behavior of the surface can
be visualized as thin compression arches in one direction and tension cables in the perpendicular direction. In reality, additional shear and bending may occur along the vicinity of the edges.

### 13.3.5 Intersection and Combination

The basic forms of latticed shells are single-curvature cylinders, double-curvature spheres, and hyperbolic paraboloids. Many interesting new shapes can be generated by intersecting and combining these basic forms. The art of intersection and combination is one of the important tools in the design of latticed shells. In order to fulfill the architectural and functional requirements, the load-resisting behavior of the structure as a whole and also its relation to the supporting structure should be taken into consideration.

For cylindrical shells, a simply way is to intersect through the diagonal as shown in Figure 13.17a. Two types of groined vaults on a square plane can be formed by combining the corresponding intersected curve surfaces as shown in Figures 13.17b and c. Likewise, combination of curved surfaces intersected from a cylinder produce a latticed shell on a hexagonal plan as shown in Figure 13.17d.

![FIGURE 13.17: Intersection and combination of cylindrical shells.](image)

For spherical shells, segments of the surface are used to cover planes other than circular, such as triangular, square, and polygonal as shown in Figure 13.18a, b, and c, respectively. Figure 13.18d shows a latticed shell on a square plane by combining the intersected curved surface from a sphere.

It is usual to combine a segment of a cylindrical shell with hemispherical shells at two ends as shown in Figure 13.19. This form of latticed shell is an ideal plan for indoor track fields and ice skating rinks.

Different solutions for assembling single hyperbolic paraboloid units to cover a square plane are shown in Figure 13.20. The combination of four equal hypar units produces different types of latticed shells supported on a central column as well as two or four columns along the outside perimeter. These basic blocks, in turn, can be added in various ways to form the multi-bay buildings.
13.4 Structural Analysis

13.4.1 Design Loads

1. Dead load—The design dead load is established on the basis of the actual loads which may be expected to act on the structure of constant magnitude. The weight of various accessories—cladding, supported lighting, heat and ventilation equipment—and the weight of the space frame comprise the total dead load. An empirical formula is suggested to estimate the dead weight $g$ of double layer grids.

$$g = \frac{1}{200} \left( \zeta \sqrt{q_w L} \right) \text{kN/m}^2$$  \hspace{1cm} (13.3)

where

- $q_w = \text{all dead and live loads acting on a double layer grid except its self-weight in kN/m}^2$
- $L = \text{shorter span in m}$
- $\zeta = \text{coefficient, 1.0 for steel tubes, 1.2 for mill sections}$

2. Live load, snow or rain load—Live load is specified by the local building code and compared with the possible snow or rain load. The larger one should be used as the design load. Each space frame is designed with a uniformly distributed snow load and further allowed for drifting depending upon the shape and slope of the structure. Often more than one assumed distribution of snow load is considered. Very little information can be found on this subject although a proposal was given by ISO for the determination of snow
loads on simple curved roofs. The intensity of snow load as specified in Basis for Design of Structures: Determination of Snow Loads on Roofs [12] is reproduced as Figure 13.21. Rain load may be important in a tropical climate especially if the drainage provisions are insufficient. Ponding results when water on a double layer grid flat roof accumulates faster than it runs off, thus causing excessive load on the roof.

3. Wind load—The wind loads usually represent a significant proportion of the overall forces acting on barrel vaults and domes. A detailed comparison of the available codes concerning wind loads has revealed quite a large difference between the practices adopted by various countries. Pressure coefficients for an arched roof springing from a ground surface that can be used for barrel vault designs are shown in Figure 13.22 and Table 13.3. For an arched roof resting on an elevated structure such as enclosure walls, the pressure coefficients are shown in Table 13.4.
The wind pressure distribution on buildings is also recommended by the European Convention for Constructional Steelwork. The pressure coefficients for an arched roof and spherical domes, either resting on the ground or on an elevated structure are presented in graphical forms as shown in Figure 13.23 and 13.24, respectively.

It can be seen that significant variations in pressure coefficients from different codes of practice exist for three-dimensional curved spaceframes. This is due to the fact that these coefficients are highly dependent on Reynolds number, surface roughness, wind velocity profile, and turbulence. It may be concluded that the codes of practice are only suitable for preliminary design purposes, especially for those important long span space structures and latticed shells with peculiar shapes. It is therefore necessary to undertake further wind tunnel tests in an attempt to more accurately establish the pressure distribution over the
rooftop surface. For such tests, it is essential to simulate the velocity profile and turbulence of the natural wind and the Reynolds number effects associated with the curved surface.

4. Temperature effect—Most space frames are subject to thermal expansion and contraction due to changes in temperature, and thus may be subject to axial loads if restrained. Potential temperature effect must be considered in the design especially when the span is comparatively large. The choice of support locations—perimeter, intermediate columns—and types of support—fixed, slid or free rotation and translation—as well as the geometry of members adjacent to the support, all contribute to minimizing the effect of thermal expansion. The temperature effect of a space frame may be calculated by the ordinary matrix displacement method of analysis and most computer programs provide such a function.

For a double layer grid, if it satisfies one of the following requirements, the calculation for temperature effect may be exempted.

(a) The joints on supports allow the double layer grid to move horizontally.

(b) Double layer grids of less than 40 m span are supported along perimeters by independent reinforced concrete columns or brick pilasters.

(c) The displacement at the top of the column due to a unit force is greater or equal to the value calculated according to the following formula:

$$\delta = \frac{L}{2\xi EA} \left( \frac{E\alpha \Delta_t}{0.05 [\sigma]} - 1 \right)$$

(13.4)
TABLE 13.4  Pressure Coefficient for an Arched Roof on an Elevated Structure

<table>
<thead>
<tr>
<th>Country code</th>
<th>Windward quarter</th>
<th>Central half</th>
<th>Leeward quarter</th>
<th>Rise/span r</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. ANSI A 58.1-1982</td>
<td>-0.9</td>
<td>-0.7 - r</td>
<td>-0.5</td>
<td>0 &lt; r &lt; 0.2</td>
</tr>
<tr>
<td></td>
<td>1.5r - 0.3</td>
<td>-0.7 - r</td>
<td>-0.5</td>
<td>0.2 ≤ r ≤ 0.3</td>
</tr>
<tr>
<td></td>
<td>2.75r - 0.7</td>
<td>-0.7 - r</td>
<td>-0.5</td>
<td>0.3 ≤ r ≤ 0.6</td>
</tr>
<tr>
<td>U.S.S.R. BC&amp;R 2.01.07-85</td>
<td>h_e/b = 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.2</td>
<td>-0.8</td>
<td>-0.4</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>-0.1</td>
<td>-0.9</td>
<td>-0.4</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>-1.0</td>
<td>-0.4</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>-1.1</td>
<td>-0.4</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>-1.2</td>
<td>-0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>China GBJ 9-87-1987</td>
<td>0.3</td>
<td>-0.8</td>
<td>-0.4</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>-0.8</td>
<td>-0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

where

- \( L \) = span of double layer grid in the direction of checking temperature effect
- \( E \) = modulus of elasticity
- \( A \) = arithmetic mean value of the cross-sectional area of members in the supporting plane (top or bottom layer)
- \( \alpha \) = coefficient of thermal expansion
- \( \Delta_t \) = temperature difference
- \( [\sigma] \) = allowable stress of steel
- \( \xi \) = coefficient, when the chords in the supporting plane are arranged in orthogonal grids \( \xi = 1 \), in diagonal grids \( \xi = 2 \), and in three-way grids \( \xi = 2 \)

5. Construction loads—During construction, structures may be subjected to loads different from the design loads after completion, depending on the sequence of construction and method of scaffoldings. For example, a space frame may be lifted up at points different from the final supports, or it may be constructed in blocks or strips. Therefore, the whole structure, or a portion of it, should be checked during various stages of construction.

13.4.2 Static Analysis

There are generally two different approaches in use for the analysis of space frames. In the first approach, the structure is analyzed directly as a general assembly of discrete members, i.e., discrete method. In the second approach, the structure is represented by an equivalent continuum like a plate or shell, i.e., continuum analogy method.

The advent of computers has radically changed the whole approach to the analysis and design of space frames. It has also been realized that matrix methods of analysis provide an extremely efficient means for rapid and accurate treatment of many types of space structures. In the matrix analysis, a structure is represented as a discrete system and all the usual equations of structural mechanics are written conveniently in matrix form. Thus, matrix analysis is particularly suitable to computer

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formulation, with an automatic sequence of operations. A number of general purpose computer programs, such as STRUDL and SAP, have been developed and are available to designers.

The two common formulations of the matrix analysis are the stiffness method and flexibility method. The stiffness method is also referred to as the displacement method because the displacements of the redundant members are treated as unknowns. The flexibility method (or force method) treats the forces in the members as unknowns. Of these two methods, the displacement method is widely used in most computer programs.

In the displacement method, the stiffness matrix of the whole structure is obtained by adding appropriately the stiffness matrixes of the individual elements. Supports are then introduced because the displacements at these points are known. A set of simultaneous equations are solved for displacements. From the joint displacements the member elongation can be found and hence the member forces and reaction at supports.

The matrix displacement method is by far the most accurate method for the analysis of space frames. It can be used without any limit on the type and shape of the structure, the loadings, the supporting conditions, or the variation of stiffness. The effect of temperature or uneven settlement of supports also can be analyzed conveniently by this method. For design work, a special purpose computer program for space frames is preferred; otherwise the input of generating nodal coordinates and member connectivity plus loading information will be a tremendous amount of work. Some

FIGURE 13.23: Wind pressure coefficients for an arched roof.
sophisticated computer programs provide the functions of automatic design, optimization, and drafting.

Double layer grids can be analyzed as pin connected and rigidity of the joints does not change the stress by more than 10 to 15%. In the displacement method, bar elements are used with three unknown displacements in x, y, and z directions at each end. For single layer reticulated shells with rigid joints, bar elements are used and the unknowns are doubled, i.e., three displacements and three rotations. Under specific conditions, single layer braced domes may be analyzed as pin connection
joints with reasonable accuracy if the rise of the dome is comparatively large and under symmetric loading.

When using a computer, the engineer must know the assumptions on which the program is based, the particular conditions for its use (boundary conditions for example), and the manner of introducing the input data. In the static analysis of the space frames, care should be taken on the following issues:

1. Support conditions—A fixed support (bolted or welded) in construction should not be treated literally as a completely fixed node in analysis. As a matter of fact, most space frames are supported on columns or walls that have a lateral flexibility. Upon the acting of external loads, there will be lateral displacements on the top of columns. Therefore, it is more reasonable to assume the support as horizontally movable rather than fixed, or as an elastic support by considering the stiffness of the supporting column.

2. Criterion for the number of reanalyzes—Usually a set of sectional areas are assumed for members and the computer will proceed to analyze the structure to obtain a set of member forces. Then the members are checked to see if the assumed areas are appropriate. If not, the structure should be reanalyzed until the forces and stiffness completely match each other. However, such extended reanalyzes by the stiffness method will induce a high concentration of stiffness and, hence, a great difference of member sections which is unacceptable for practical use. Therefore, it is necessary to limit the number of reanalyzes. In practice, certain criteria are specified such that the reanalysis will terminate automatically. One of the criterion is suggested as the number of the modified members less than 5% of the total number of members. Usually three or four runs will produce a satisfactory result.

3. Checking of computer output—It is dangerous for an engineer to rely on the computer output as being infallible. Always try to estimate and anticipate results. A simple manual calculation by approximate method and comparing it with computer output will be beneficial. By doing so, an order of magnitude for the results can be obtained. In this operation, intuition also plays an important role. At the same time, simple checks should be done to test the reliability of the computer program, such as the equilibrium of forces at nodes and the equilibrium of total loading with the summation of reactions. A check on the deflections along certain axes of the structure would also be helpful. The size and location of any large deflection should be noted. All deflections should be scanned to look for possible bad solutions caused by improper modeling of the structure. This check is made easily if the program has the ability to produce a deformed geometry plot.

A continuum analogy method may also be used for the static analysis of space frames. This is to replace a latticed structure by an equivalent continuum which exhibits equivalent behavior with respect to strength and stiffness. The equivalent rigidity is used for the stress and displacement analysis in the elastic range, and particularly so for stability and dynamic analysis. It is useful as well in order to provide an understanding of the overall behavior of the structure by large. By using equivalent rigidity, the thickness, elastic moduli, and Poisson’s ratio are determined for the equivalent continuum, and the fundamental equations that govern the behavior of the equivalent continuum are established as in the usual continuum theory. Therefore, the methods of solution and the results of the theory of plates and shells are directly applicable. Thus, certain types of latticed shells and double layer grids can be analyzed by treating them as a continuum and applying the shell or plate analogy. This method has been found to be satisfactory where the loading is uniform and the load transfer is predominantly through membrane action.

Some difficulties may occur in the application of the continuum analogy method. The boundary conditions of the continuum cannot be entirely analogous to the boundary condition of the discrete
prototype. Also, some of the effects that are relatively unimportant in the case of continua may be significant in the case of space frames. Two of these merit mention. The effect of shear deformation in elastic plates and shells is essentially negligible, whereas the contributions of web members connecting the layers of a space frame can be significant to the total deformation. Similarly, the correct continuum model of a rigidly connected space frame must allow for the possibility of rotation of joints independent of the rotations of normal sections. Such models are more complex than the usual ones, and few solutions of the governing equations exist.

It is useful to compare the discrete method and continuum analogy method. The continuum analogy method can only be applied to regular structures while the discrete method can handle arbitrary structural configuration. The computational time is much less for an equivalent shell analysis than a stiffness method analysis. The work involved in a continuum analogy method includes calculating the equivalent rigidity, the forces in the equivalent continuum, and finally the forces in the members. This will go through a discrete-continuum-discrete process and, hence, involves further approximation. To summarize, the continuum analogy method is most valuable at the stage of conceptual and preliminary design while the discrete method should be used for a working design.

13.4.3 Earthquake Resistance

One of the important issues that must be taken into consideration in the analysis and design of space frames is the earthquake excitation in case the structure is located in a seismic area. The response of the structure to earthquake excitation is dynamic in nature and usually a dynamic analysis is necessary. The analysis is complicated due to the fact that the amplitude of ground accelerations, velocities, and motions is not clearly determined. Furthermore, the stiffness, mass distribution, and damping characteristics of the structure will have a profound effect on its response: the magnitude of internal forces and deformations.

The dynamic behavior of a space frame can be studied first through the vibration characteristics of the structure that is represented by its natural frequencies. The earthquake effects can be reflected in response amplification through interaction with the natural dynamic characteristics of the structures. Thus, double layer grids can be treated as a pin-connected space truss system and their free vibration is formulated as an equation of motion for a freely vibrating undamped multi-degree-of-freedom system. By solving the generalized eigenvalue problems, the frequencies and vibration modes are obtained.

A series of double layer grids of different types and spans were taken for dynamic analysis [23]. The calculation results show some interesting features of the free vibration characteristics of the space frames. The difference between the frequencies of the first 10 vibration modes is so small that the frequency spectrums of space frames are rather concentrated. The variation of any design parameter will lead to the change of frequency. For instance, the boundary restraint has a significant influence on the fundamental period of the space frame: the stronger the restraint, the smaller the fundamental period.

The fundamental periods of most double layer grids range from 0.37 to 0.62 s which are less than that of planar latticed trusses of comparable size. This fact shows clearly that the space frames have relatively higher stiffness. Investigating into the relation of fundamental periods of different types of double layer grids with span, it is found that the fundamental period increases with the span, i.e., the space frames will be more flexible for longer span. The response of space frames with shorter span will be stronger.

The vibration modes of double layer grids could be classified mainly as vertical modes and horizontal modes that appear alternately. In most cases, the first vibration modes are vertical. The vertical modes of different types of double layer grids demonstrate essentially the same shape and the vertical frequencies for different space frames of equal span are very close to each other. It was found that the forces in the space frame due to vertical earthquake are mainly contributed by the
first three symmetrical vertical modes. Certain relations could be established between the first three frequencies of the vertical mode as follows:

\[
\omega_{n2} = (2 - 3.5) \omega_{n1} \quad (13.5)
\]
\[
\omega_{n3} = (4 - 4.6) \omega_{n2} \quad (13.6)
\]

where \(\omega_{n1} \), \(\omega_{n2} \), and \(\omega_{n3} \) are the first, second, and third vertical frequencies, respectively.

The simplest way to estimate the earthquake effect is a quasi-static model in which the dynamic action of the ground motion is simulated by a static action of equivalent loads. The manner in which the equivalent static loads are established is introduced in many seismic design codes of different countries. In the region where the maximum vertical acceleration is 0.05 \(g\), usually the earthquake effect is not the governing factor in design and it is not necessary to check the forces induced by vertical or horizontal earthquake. In the area where the maximum vertical acceleration is 0.1 \(g\) or greater, a factor of 0.08 to 0.2, depending on different codes, is used to multiply the gravitational loads to represent the equivalent vertical earthquake load. It should be noticed that in certain seismic codes, the live load that forms a part of gravitational loads is reduced by 50%. The values of vertical seismic forces in the members of double layer grids are higher near the central region and decrease gradually towards the perimeters. Thus, the ratio between the forces in each member due to vertical earthquake and static load is not constant over the whole structure. The method of employing equivalent static load serves only as an estimation of the vertical earthquake effect and provides an adequate level of safety.

Due to the inherent horizontal stiffness of double layer grids, the forces induced by horizontal earthquake can be resisted effectively. In the region of 0.1 \(g\) maximum acceleration, if the space grids are supported along perimeters with short or medium span, it is not required to check the horizontal earthquake. However, for double layer grids of longer span or if the supporting structure underneath is rather flexible, seismic analysis in the horizontal direction should be taken. In the case of latticed shells with a curved surface, the response to horizontal earthquake is much stronger than double layer grids depending on the shape and supporting condition. Even in the region with maximum vertical acceleration of 0.05 \(g\), the horizontal earthquake effect on latticed shells should be analyzed. In such analysis, coordinating the action of the space frame and the supporting structure should be considered. A simple way of coordination is to include the elastic effect of the supporting structure. This is represented by the elastic stiffness provided by the support in the direction of restraint. The space frame is analyzed as if the supports have horizontally elastic restraints. For a more accurate analysis, the supporting columns are taken as member with bending and axial stiffness and analyzed together with the space frame. In the analysis, it is also important to include the inertial effect of the supporting structure, which has influence on the horizontal earthquake response of the space frame.

In the case of more complex structures or large spans, dynamic analysis, such as the response spectrum method for modal analysis, should be used. Such method gives a good estimate of the maximum response during which the structure behaves elastically. For space frames, the vertical seismic action should be considered. However, few recorded data on the behavior of such structures under vertical earthquake exist. In some seismic design codes, the magnitude of the vertical component may be taken as 50 to 65% of the horizontal motions. Use of 10 to 20 vibration modes is recommended for the space frames when applying the response spectrum method.

For space frames with irregular and complicated configurations or important long span structures, the time-history analysis method should be used. The number of acceleration records or synthesized acceleration curves for the time-history analysis is selected according to intensity, location of earthquake, and site category. In usual practice, at least three records are used for comparison. Such a method is an effective tool to calculate the earthquake response when large, inelastic deformations are expected.

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The behavior of latticed structures under dynamic loads or, more specifically, the performance of latticed structures due to earthquake, was the main concern of structural engineers. An ASCE Task Committee on Latticed Structures Under Extreme Dynamic Loads was formed to investigate this problem. One of the objectives was to determine if dynamic conditions have historically been the critical factor in failure of lattice structures. A short report on “Dynamic Considerations in Latticed Structures” was submitted by the Task Committee in 1984. Eight major failures of latticed roof structures were reported but notably none of them was due to earthquake.

Since the ASCE report was published, valuable information on the behavior of space frames during earthquake has been obtained through two seismic events. In 1995, the Hanshin area of Japan suffered a strong earthquake and many structures were heavily damaged or destroyed. However, when compared with other types of structures, most of the damage to space frames located in that area was relatively minor [13]. It is worthwhile to mention that two long span sport arenas of space frame construction were built on an artificial island in Kobe and no major structural damage was found. On the other hand, serious damage to a latticed shell was found on the roof structure of a hippodrome stand where many members were buckled. The cause of the damage is not due to the strength of the space frame itself but the failure of the supporting structure. Another example of serious damage to double layer grids for the roof structure of a theater occurred in 1985 when a strong earthquake struck the Kashigor District of Sinkiang Uygur Autonomous Region in China [15]. Failure was caused by a flaw in the design as the elastic stiffness and inertial effect of supporting structures were completely ignored. Behavior of space frame structures under a strong earthquake has generally been satisfactory from a strength point of view. Experiences gained from strong earthquakes shows the space frames demonstrate an effective spatial action and consequently a reasonably good aseismic behavior.

### 13.4.4 Stability

Although a great amount of research has been carried out to determine the buckling load of latticed shells, the available solutions are not satisfactory for practical use. The problem is complicated by the effect of geometric nonlinearity of the structure and also the influence of the joint system according to which the members can be considered as pin-connected or partially or completely restrained at the nodes.

The following points are important in the buckling analysis of latticed shells [7]:

1. Decision on which kind of nonlinearity is necessary to be used—only geometrical nonlinearity with the elastic analysis, or geometrical and material nonlinearities with the elastic-plastic analysis.
2. Choosing the physical model—equivalent continuum or discrete structure.
3. Choosing the computer model and numerical procedure for tracing the non-linear response for precritical behavior, collapse range, and post-critical behavior.
4. Study of factors influence load carrying capacity—buckling modes, density of network, geometrical and mechanical imperfections, plastic deformations, rigidity of joints, load distributions, etc.
5. Experimental investigations to provide data for analysis (rigidity of joints, postbuckling behavior of individual member, etc.) and confirmation of theoretical values.

Generally speaking, there are three types of buckling that may occur in latticed shells:

1. member buckling (Figure 13.25a)
2. local or dimple buckling at a joint (Figure 13.25b)
3. general or overall buckling of the whole structure (Figure 13.25c)
Member buckling occurs when an individual member becomes unstable, while the rest of the space frame (members and nodes) remain unaffected. The buckling load $P_{cr}$ of a straight prismatic bar under axial compression is given by

$$P_{cr} = \frac{\pi^2 E_e I}{D^2} \alpha (c_i, c_j, w_o, e, m)$$

where

- $E_e$ = effective modulus of elasticity that coincides with Young’s modulus in the elastic range
- $I$ = moment of inertia of member
- $D$ = length of the member

The coefficient $\alpha$ takes different values depending on the parameter in the parentheses. The quantities $c_i$ and $c_j$ characterize the rotational stiffness of the joints, $w_o$ is the initial imperfection, $e$ is the eccentricity of the end compressive forces, and $m$ is the end shear forces and moments. A reduced length $l_o$ should be used in place of $l$ when the ratio of the joint diameter to member length is relatively large. On the basis of Equation 13.7, the design code for steel structures in different countries provides methods for estimating member buckling, usually by introducing the slenderness ratio $\lambda = l/r$, where $r$ is the radius of gyration of the member’s section.

The local buckling of a space frame consists of a snap-through buckling which takes place at one joint. Snap-through buckling is characterized by a strong geometrical non-linearity. Local buckling is apt to occur when the ratio of $l/R$ (where $l$ is the equivalent shell thickness and $R$ is the radius of curvature) is small. Similarly, local buckling of a space frame is likely to occur in single layer latticed shells.

Local buckling is greatly affected by the stiffness of and the loads on the adjacent members. Consider the pin-connected structure shown in Figure 13.26. Buckling load $q_{cr}$ in terms of uniform normal load per unit area can be expressed as

$$\frac{AEI}{12R^3} \leq q_{cr} \leq \frac{AEI}{6R^3}$$

where

- $A$ = cross-sectional area of the member
- $E$ = modulus of elasticity
- $R$ = radius of an equivalent spherical shell through points B-A-B

In practice, different types of joints used in the design will possess different flexural strength; thus, the actual behavior of the joint and member assembly should be incorporated in determining the local buckling load. An approximate formula was proposed by Lind [16] and is applicable to triangular networks having all elements of the same cross-sections. For the uniform load, the critical load is

$$Q_{cr} = \frac{E}{1 + \alpha^2 / 8\pi^2} \left( 0.47 \frac{Al^3}{R^3} + 3 \frac{BI}{1R} \right)$$
where

\[ E_t = \text{tangent modulus of elasticity} \]
\[ R = \text{radius of curvature of the framework mid-surface} \]
\[ r = \text{radius of gyration} \]
\[ B = \text{non-dimensional bending stiffness of the grid given in Table 13.5} \]

For the concentrated load, the following two formulae are presented

\[ W_{cr} = \frac{3EAh^3}{l^3} \left[ \frac{8B}{\alpha^2} + 0.241 \left( 1 - 5.95 \frac{8B}{\alpha^2} \right) \right] \quad (13.10) \]

which is valid for \( \alpha > 9 \), and

\[ W_{cr} = 0.0905EA \left( \frac{1}{R} \right)^3 \quad (13.11) \]

for a regular pin-jointed triangular network.

TABLE 13.5  Equivalent Bending Stiffness B

| \( \alpha \) | 1/32 | 1/16 | 1/8 | 1/4 | 1/2 | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| B | 0.866 | 0.873 | 0.886 | 0.900 | 1.176 | 1.85 | 3.35 | 4.83 | 6.48 | 7.35 | 7.80 | 7.90 |

The overall buckling occurs when a relatively large area of the space frame becomes unstable, and a relatively large number of joints is involved in the buckle. For most cases, in overall buckling of a space frame, the wave length is significantly greater than the member length. Local buckling often plays the role of trigger for overall buckling.

The type of buckling collapse of a space frame is greatly influenced by the following factors: its Gaussian curvature, whether it is a single or double layer system, the degree of statical indeterminacy, and the manner of supporting and loading. Generally speaking, a shallow shell of positive Gaussian
curvature, like a dome, is more prone to overall buckling than a cylindrical shell of zero Gaussian curvature. Recent research reveals that a hyperbolic paraboloid shell is less vulnerable to overall buckling and the arrangement of the grids has a considerable influence on the stability and stiffness of the shell. It is best to arrange the members along the direction of compressive forces. A single layer space frame exhibits greater sensitivity to buckling than a double layer structure. Moreover, various types of buckling behavior may take place simultaneously in a complicated relation. For double layer grids, in most cases, it is sufficient to examine the member collapse which may occur in the compressive chord members.

The theoretical analysis of buckling behavior may be approached by two methods: continuum analogy analysis and discrete analysis. Since almost all space frames are constructed from nearly identical units arranged in a regular pattern, it is generally accepted that the analysis on the basis of the equivalent continuum serves as an important tool in the investigation of the buckling behavior of space frames. Numerous analytical and experimental studies on the buckling of continuous shells have been performed and the results can be applied to the latticed shells.

The buckling formula for a spherical shell subjected to a uniformly distributed load normal to the middle surface can be expressed as

\[ q_{cr} = k E \left( \frac{t}{R} \right)^2 \]  

where \( t \) is the thickness and \( R \) is the radius of the shell.

Different values of the coefficient \( k \) were obtained by various investigators.

- \( k = 1.21 \) (Zoelly [1915], based on classical linear theory)
- \( k = 0.7 \) (experiments on very carefully prepared models)
- \( k = 0.366 \) (Karman and Tsien [1939], based on nonlinear elastic theory)
- \( k = 0.228, 0.246 \) (del Pozo [1979], for \( \mu = 0 \) and 0.3, respectively)

For a triangulated dome where an equivalent thickness is used, Wright [22] derived the formula by using

\[ E = \frac{AE}{3rl} \quad t = 2\sqrt{3}r \quad k = 0.4 \]

\[ q_{cr} = 1.6E \frac{Ar}{lR^2} \]  

(13.13)

The critical load for overall buckling may also be expressed as the following formula for comparison

\[ q_{cr} = k' \frac{E}{R^2} \left( \frac{t_m}{t_b} \right)^{1/2} \left( \frac{t_m}{t_b} \right)^{1/2} \]  

(13.14)

where

- \( t_m \) = effective in-plane thickness
- \( t_b \) = effective bending thickness
- \( k' \) = 0.377 [22]
- \( k' = 0.365 \)
- \( k' = 0.247 \)
- \( k' = 0.294 \) [8]

Discrete analysis is a more powerful tool to study the whole process of instability for space frames. As shown in Figure 13.27, a structure may lose its stability when it has reached a "limit point", where the stiffness is lost completely. On the other hand, a structure such as a dome may lose its stability by a sudden buckling into a mode of deformation before the limit point, which occurs at a distinct critical point— "bifurcation point" on the load path. It should be noted that the initial imperfection of the structure will greatly reduce the value of critical load, and certain types of space frame are very sensitive to the present of imperfection.
In the stability analysis, usually the characteristic at certain special states is investigated, i.e., the stability mode and critical load are analyzed as an eigenvalue problem. Researchers are now more interested in studying the whole process of nonlinear stability. As a result of the development of the computer matrix method, numerical analyses of large systems have become straightforward. Therefore, the discrete analysis of a space frame, itself a discrete structure, is very suitable for the study of stability problems. Major problems encountered in the nonlinear stability process are: the mathematical and mechanical modeling of the structure, the numerical technique for solving nonlinear equations, and the tracing method for the nonlinear equilibrium path. Much research has been carried out in the above area.

The Newton-Raphson method or the modified Newton-Raphson method is the fundamental method for solving the nonlinear equilibrium equations and has proved to be one of the most effective methods. The purpose of tracing the nonlinear equilibrium path is as follows: (1) to provide equilibrium analysis for the pre-buckling state, (2) to determine the critical point, such as the limit point or bifurcation point on the load path and its critical load, (3) to trace the post-buckling response. On the basis of the increment-iteration process for the finite element method, techniques for the analysis of nonlinear equilibrium path and its tracing tactics have made significant progress in recent years. Numerical methods used for the construction of equilibrium paths associated with nonlinear problems, such as the load incremental method, constant arc-length method, displacement control method, etc. were developed by different authors. Because each of the techniques has its advantages and disadvantages in the derivation of fundamental equations, accuracy of solution, computing time, etc., the selection of the appropriate method has a profound influence on the efficiency of the computation. In the present stage of development, complicated equilibrium paths can be traced with the aid of the above technique. Computer programs have been developed for the whole process of nonlinear stability and can be used for the design of various types of latticed shells.

13.5 Jointing Systems

13.5.1 General Description

The jointing system is an extremely important part of a space frame design. An effective solution of this problem may be said to be fundamental to successful design and construction. The type of jointing depends primarily on the connecting technique, whether it is bolting, welding, or applying special mechanical connectors. It is also affected by the shape of the members. This usually involves a different connecting technique depending on whether the members are circular or square hollow.
sections or rolled steel sections. The effort expended on research and development of jointing systems has been enormous and many different types of connectors have been proposed in the past decades.

The joints for the space frame are more important than the ordinary framing systems because more members are connected to a single joint. Furthermore, the members are located in a three-dimensional space, and hence the force transfer mechanism is more complex. The role of the joints in a space frame is so significant that most of the successful commercial space frame systems utilize proprietary jointing systems. Thus, the joints in a space frame are usually more sophisticated than the joints in planar structures, where simple gusset plates will suffice.

In designing the jointing system, the following requirements should be considered. The joints must be strong and stiff, simple structurally and mechanically, and yet easy to fabricate without recourse to more advanced technology. The eccentricity at a joint should be kept to a minimum, yet the joint detailing should provide for the necessary tolerances that may be required during the construction. Finally, joints of space frames must be designed to allow for easy and effective maintenance.

The cost of the production of joints is one of the most important factors affecting the final economy of the finished structure. Usually the steel consumption of the connectors will constitute 15 to 30% of the total. Therefore, a successful prefabricated system requires joints that must be repetitive, mass produced, simple to fabricate, and able to transmit all the forces in the members interconnected at the node.

All connectors can be divided into two main categories: the purpose-made joint and the proprietary joint used in the industrialized system of construction. The purpose-made joints are usually used for long span structures where the application of standard proprietary joints is limited. An example of such types of joints is the cruciform gusset plate for connecting rolled steel sections as shown in Figure 13.28.

![FIGURE 13.28: Connecting joint with cruciform gusset plate.](image)

A survey around the world will reveal that there are over 250 different types of jointing systems suggested or used in practice, and there are some 50 commercial firms trying to specialize in the manufacture of proprietary jointing systems for space frames. Unfortunately, many of these systems have not proved attainment of great success mainly because of the complexity of the connecting method. Tables 13.6 through 13.8 give a comprehensive survey of the jointing systems all over the world. All the connection techniques can be divided into three main groups: (1) with a node, (2) without a node, and (3) with prefabricated units.

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<th>Node</th>
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<td></td>
<td></td>
<td>Octaplate, NL</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Piramodul large span, NL</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nodas, U.K.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 13.6  Connection Types with a Node (continued)

<table>
<thead>
<tr>
<th>Node</th>
<th>Connector</th>
<th>Member</th>
<th>Cross-section</th>
<th>Examples</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td></td>
<td></td>
<td></td>
<td>Sahale, Germany</td>
<td>A41</td>
</tr>
<tr>
<td>Prism</td>
<td></td>
<td></td>
<td></td>
<td>Mero BK, Germany</td>
<td>A42</td>
</tr>
<tr>
<td>Hollow</td>
<td></td>
<td></td>
<td></td>
<td>Mero TK and ZK, Germany</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mero NK, Germany</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Satterwhite, U.S.</td>
<td></td>
</tr>
</tbody>
</table>


13.5.2 Proprietary System

Some of the most successful prefabricated jointing systems are summarized in Table 13.9. This is followed by a further description of each system.

1. Mero

The Mero connector, introduced some 50 years ago by Dr. Mengeringhausen, proved to be extremely popular and has been used for numerous temporary and permanent buildings. Its joint consists of a node that is a spherical hot-pressed steel forging with flat facets and tapped holes. Members are circular hollow sections with cone-shaped steel forgings welded at the ends which accommodate connecting bolts. Bolts are tightened by means of a hexagonal sleeve and dowel pin arrangement, resulting in a completed joint such as that shown in Figure 13.29. Up to 18 members can be connected at a joint with no eccentricity. The manufacturer can produce models of different size with diameter ranging from 46.5 mm to 350 mm, the corresponding bolts ranging from M 12 to M 64 with a maximum permissible force of 1413 kN. A typical space-module of a Mero system is a square pyramid (1/2 Octahedron) with both chord and diagonal members of the same length "a", angles extended are 90° or 60°. Thus, the depth of the space-module is \( a/\sqrt{2} \) and the vertical angle between diagonal and chord member is 54.7°.

The Mero connector has the advantage that the axes of all members pass through the center of the node, eliminating eccentricity loading at the joint. Thus, the joint is only under the axial forces. Then tensile forces are carried along the longitudinal axis of the bolts and resisted by the tube members through the end cones. The compressive forces do not produce any stresses in the bolts; they are distributed to the node through the hexagonal sleeves. The size of the connecting bolt of compression members based on the diameter calculated from its internal forces may be reduced by 6 to 9 mm.

The diameter of a steel node may be determined by the following equations (Figure 13.29).

\[
D \geq \sqrt{\frac{d_2}{\sin \theta} + (d_1 \cdot \cos \theta + 2 \xi \cdot d_1)}^2 + \eta^2 d_1^2 \tag{13.15}
\]

However, in order to satisfy the requirements of the connecting face of the sleeve, the diameter should be checked by the following equation:
TABLE 13.7  Connection Types without a Node

<table>
<thead>
<tr>
<th>Node</th>
<th>Connector</th>
<th>Member</th>
<th>Cross-section</th>
<th>Examples</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form of member</td>
<td>Forming</td>
<td></td>
<td></td>
<td>Backminister Fuller, Notadome, NL.</td>
<td>B11</td>
</tr>
<tr>
<td>Flattened and bending</td>
<td></td>
<td></td>
<td></td>
<td>Radial, Australia, Harley, Australia</td>
<td>B12</td>
</tr>
<tr>
<td>Addition of member</td>
<td>Plate(s)</td>
<td></td>
<td></td>
<td>Mai Sky, U.S.</td>
<td>B21</td>
</tr>
<tr>
<td>Member end</td>
<td></td>
<td></td>
<td></td>
<td>Pieter Hoybers, NL, Pierce, U.S., Backminister Fuller</td>
<td>B22</td>
</tr>
</tbody>
</table>


\[
D \geq \sqrt{\left(\frac{\eta d_2}{\sin \theta} + \eta d_1 \cot \theta \right)^2 + \eta^2 d_1^2} \tag{13.16}
\]

where

- \( D \) = diameter of steel ball (mm)
- \( \theta \) = the smaller intersecting angle between two bolts (rad)
- \( d_1d_2 \) = diameter of bolts (mm)
- \( \xi \) = ratio between the inserted length of the bolt into the steel ball and the diameter of the bolt
- \( \eta \) = ratio between the diameter of the circumscribed circle of the sleeve and the diameter of the bolt

\( \xi \) and \( \eta \) may be determined, respectively, by the design tension values or compression strength of bolt. Normally \( \xi = 1.1 \) and \( \eta = 1.8 \).

The diameter of a steel ball should be taken as the larger value calculated from the above two equations.

The Mero connector was originally developed for double layer grids. Due to the increasing usage of non-planar roof forms, it is required to construct the load-bearing space frame integrated with cladding element. A new type of jointing system called Mero Plus System was developed so that a variety of curved and folded structures are possible. Square or rectangular hollow sections are used to match the particular requirements of the cladding so that a flush transition from member to connecting node can be executed. The connector can transmit shear force, resist torsion, and in special cases can resist bending moment. There are four groups in this system which are described as follows.

(a) Disc Node (Type TK) (Figure 13.31)—This is a planar ring-shaped node connecting
TABLE 13.8 Connection Types with Prefabricated Units

<table>
<thead>
<tr>
<th>Node</th>
<th>Prefabricated unit</th>
<th>Member cross-section</th>
<th>Examples</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric solid</td>
<td></td>
<td></td>
<td>Space deck, U.K.</td>
<td>C11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mero DL, Germany</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Unistrut, France</td>
<td></td>
</tr>
<tr>
<td>2D components</td>
<td></td>
<td></td>
<td>nameless system,</td>
<td>C12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Italy</td>
<td></td>
</tr>
<tr>
<td>3D components</td>
<td></td>
<td></td>
<td>nameless system,</td>
<td>C22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Italy</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cabic, U.K.</td>
<td>C31</td>
</tr>
</tbody>
</table>


TABLE 13.9 Commonly Used Proprietary Systems

<table>
<thead>
<tr>
<th>Name</th>
<th>Country</th>
<th>Period of development</th>
<th>Material</th>
<th>Connecting method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mero</td>
<td>Germany</td>
<td>1940–1950</td>
<td>Steel</td>
<td>Bolting</td>
</tr>
<tr>
<td>Triodetic</td>
<td>Canada</td>
<td>1950–1960</td>
<td>Aluminum</td>
<td>Inserting member</td>
</tr>
<tr>
<td>Unistrut</td>
<td>U.S.</td>
<td>1950–1960</td>
<td>Steel</td>
<td>Bolting</td>
</tr>
<tr>
<td>Unibat</td>
<td>France</td>
<td>1960–1970</td>
<td>Steel</td>
<td>Bolting</td>
</tr>
<tr>
<td>NS</td>
<td>Japan</td>
<td>1970–1980</td>
<td>Steel</td>
<td>Bolting</td>
</tr>
</tbody>
</table>

5 to 10 members of square or rectangular sections. A single bolt is used to connect the node and member and depth of the node is equal to the member section depth. Such jointing systems can transmit shear force and resist rotation. In the following discussion, the U-angle is designated as the angle between two members connected to the same node. Also, the V-angle is the angle between the member axis and the normal in the plane of the node which is a measure of curvature. For a disc node, the U-angle varies from 30° to 80° and the V-angle varies from 0° to 10°. This type of jointing system is essentially pin-jointed connections and is suitable for latticed shells made of triangular meshes.

(b) Bowl Node (Type NK) (Figure 13.32)—This is a hemispherical node connecting top chord and diagonal members. Single bolted connection from node to member is used. The top chord members of square or rectangular sections can be loaded in shear and are fitted flush to the nodes. Bowl nodes are used for double layer planar and curved surfaces, in particular buildings irregular in plan or pyramid in shape.
FIGURE 13.29: Mero system.

FIGURE 13.30: Dimensions of spherical node.
The diagonals and lower chords are constructed in an ordinary Mero system with circular tubes and spherical nodes.

(c) Cylinder Node (Type ZK) (Figure 13.33) — This is a cylindrical node with a multiple bolted connection that can transmit bending moment. Usually the node can connect 5 to 10 square or rectangular sections that can take transverse loading. Connection angle varies: $30^\circ$ to $100^\circ$ for V-angle, $0^\circ$ to $10^\circ$ for V-angle. Cylinder nodes are used in singly or doubly curved surface of latticed shells with trapezoidal meshes where flexural rigid connections are required.

(d) Block Node (Type BK) (Figure 13.34) — This is a block- or prism-shaped solid node connecting members of square or rectangular sections. The U-angle varies from $70^\circ$ to $120^\circ$ and V-angle varies from $0^\circ$ to $10^\circ$. It can be used for singly or doubly curved surfaces with pin-jointed or rigid connections where the number of members is small. The structure is of simple geometry and small dimensions.

2. Space Deck
The Space Deck system, introduced in England in the early fifties, utilizes pyramidal units that are fabricated in the shop, as shown in Figure 13.35. The four diagonals made of rods or bars are welded to the corners of the angle frame and joined to a fabricated boss.
at the apex. It is based on square pyramid units that form a configuration of square on square offset double layer space grids. The units are field-bolted together through the angle frames. The apexes of the units are connected in the field by using tie bars made from high-tensile steel bars. Camber can be achieved by adjusting the tie bar lengths, since right-hand and left-hand threading is provided in the boss. The Space Deck system is usually used for buildings of span less than 40 m with a standard module and depth of 1.2 m. A minimum structural depth of 0.75 m is also provided. For higher design loading and larger spans, alternative production modules of 1.5 m and 2.0 m with the same depth as the module are also available.

3. Triodetic

The joint for the Triodetic system, developed in Canada, consists of an extruded aluminum connector hub with serrated keyways. Each member end is pressed in order to form a coined edge that fits into the hub keyway. The joint is completed when the members are inserted into the hub, washers are placed at each end of the hub, and a screw bolt is passed through the center of hub, as shown in Figure 13.36. The Triodetic connector can be used for any type of three-dimensional space frame. Originally only aluminum structures were built in this system, but later space frames were erected using galvanized steel tubes and aluminum hubs. Triodetic double layer grids have been used up to 33 m clear span. The basic module can be almost any size up to approximately 2.7 m in square. The depth is usually 70% of the module size.
4. **Unistrut**
   The Unistrut system was developed in the U.S. in the early fifties. Its joint consists of a connector plate that is press-formed from steel plate. The members are channel-shape cold-formed sections and are fastened to the connector plate by using a single bolt at each end. The connectors for the top and bottom layers are identical and therefore the Unistrut double layer grids consist of four components only, i.e., the connector plate, the strut, the bolt, and the nut (see Figure 13.37). The maximum span for this system is approximately 40 m with standard modules of 1.2 m and 1.5 m. The name of Moduspan has also been used for this system.

5. **Oktaplatte**
   The Oktaplatte system utilizes hollow steel spheres and circular tube members that are connected by welding. The node is formed by welding two hemispherical shells together which are made from steel plates either by hot or cold pressing. The hollow sphere may be reinforced with an annular diaphragm. This type of node was popular at the early stage of development of space frames. It is also useful for the long span structures where other proprietary systems are limited by their bearing capacity. Hollow spheres with diameter up to 500 mm have been used. It can be applied to single layer latticed shells as the joint can be considered as semi- or fully rigid. The whole jointing system and the hollow sphere...
FIGURE 13.34: Block node (Type BK).

The allowable bearing strength of hollow spheres can be calculated by the following empirical formulas:

Under compression

\[ N_c = \eta_c \left( 6.6td - 2.2 \frac{t^2d^2}{D} \right) \frac{1}{K} \text{ (tons)} \]  \hspace{1cm} (13.17)

Under tension

\[ N_t = \eta_t (0.6td\pi) \sigma \]  \hspace{1cm} (13.18)

where

- \( D \) = diameter of hollow sphere (cm)
- \( t \) = wall thickness of hollow sphere (cm)
- \( d \) = diameter of the tubular member (cm)
allowable tensile stress

amplification factors due to the strengthening effect of the diaphragm, taken as 1.4 and 1.1, respectively

factor of safety

6. Unibat

The Unibat system, developed in France, consists of pyramidal units by arranging the top layer set on a diagonal grid relative to the bottom layer. The short length of the top chord members results in less material being required in these members to resist applied compressive and bending stresses. The standard units are connected to the adjacent units by means of a single high-tensile bolt at each upper corner. The apex and corners of the pyramidal unit may be forgings, to which the top chord and web members are
welded. The units may employ any combination of rolled steel or structural sections. As shown in Figure 13.39, the top chords are rolled I sections and web members are square hollow sections. The bottom layer is formed by a two-way grid of circular hollow sections which are interconnected with the apex by a single vertical bolt. Numerous multi-story buildings, as well as large span roofs over sports buildings have been built using the Unibat system since 1970.

7. Nodus

The Nodus system was developed in England in the early seventies. Its joint consists of
half-casings which are made of cast steel and have machined grooves and drilled holes, as shown in Figure 13.40. The chord connections are made of forged steel and have machined teeth, and are full-strength welded to the member ends. The teeth and grooves have an irregular pitch in order to ensure proper engagement. The forked connectors are made of cast steel and are welded to the diagonal members. For the completed joint, the centroidal axes of the diagonals intersect at a point that generally does not coincide with the corresponding intersecting points of the chord members. This eccentricity produces some amount of local bending in the chord members and the joint components. Destructive load tests performed on typical joints usually result in failures due to bending of the teeth in the main half-casing. The main feature of the Nodus jointing system is that all fabrication is carried out in the workshop so that only the simplest erection techniques are necessary for the assembly of the structure on-site.

8. NS Space Truss
The NS Space Truss system was introduced around 1970 by the Nippon Steel Corporation. It originates from the space truss technology developed for the construction of the huge roof at the symbol zone for Expo '70 in Japan. The NS Space Truss system has a joint consisting of thick spherical steel shell connectors open at the bottom for bolt insertion. The structural members are steel hollow sections having specially shaped end cones welded to both ends of the tube. End cones have threaded bolt holes. Special high strength bolts are used to join the tubular members to the spherical shell connector. The NS nodes enable several members to be connected to one node from any direction without any eccentricity of internal forces. The NS Space Truss system has been used successfully for many large span double and triple layer grids, domes, and other space structures. The connection detail of the NS node is shown in Figure 13.41.

13.5.3 Bearing Joints
Space frames are supported on columns or ring beams through bearing joints. These joints should posses enough strength and stiffness to transmit the reactions at the support safely. Under the vertical loading, bearing joints are usually under compression. In some double layer grids with
diagonal layout, bearing joints at corners may resist tension. In latticed shells, both vertical and horizontal reactions are acting on the bearing joints. The restraint of a bearing joint has a distinct influence on the joint displacement and member forces. The construction detail of a bearing support should conform to the restraint assumed in the
design as near as possible. If such requirement is not satisfied, the magnitude or even the sign of the member forces may be changed.

The axes of all connecting members and the reaction should be intersected at one point at the support where a hinged joint is used. This will allow a free rotation of the joint. From an engineering standpoint, the space frame may be fixed in the vertical direction. While in the horizontal direction, it may be fixed either tangential or normal to the boundary or both. The way that the space frame is fixed often depends on the temperature effect. If the bearing support can allow a horizontal motion normal to the boundary, then the member forces due to the temperature variation can be neglected. In such case, the bearing should be constructed so that it can slide horizontally. For those space frames with large spans or complicated configurations, especially curved surface structures supported on sloped base, care should be exercised to ensure a reliable bearing support.

Typical details for bearing joints are shown in Figure 13.42. The simplest form of bearings is to establish the joint on a flat plate and anchored by bolts as shown in Figure 13.42a or b. This joint seems to be fixed at the support, but in structural analysis it has to be incorporated with the supporting structure, such as columns or walls that have a lateral flexibility. Figure 13.42c shows the joint is resting on a curved bearing block which allows rotation along the curved surface. Such type of construction can be considered as a hinged joint. If a laminated elastomeric pad is used under the joint as shown in Figure 13.42d, a new type of bearing joint is formed. Due to the shear deformation of the elastomeric pad, the joint can produce both rotation and horizontal movements. It is very effective to accommodate the horizontal deformation caused by temperature variation or earthquake action.

### 13.6 Defining Terms

- **Aspect ratio**: Ratio of longer span to shorter span of a rectangular space frame.
- **Braced (barrel) vault**: A space frame composed of member elements arranged on a cylindrical surface.
- **Braced dome**: A space frame composed of member elements arranged on a spherical surface.
- **Continuum analogy method**: A method for the analysis of a space frame where the structure is analyzed by assuming it as an equivalent continuum.
- **Depth**: Distance between the top and bottom layer of a double layer space frame.
- **Discrete method**: A method for the analysis of a space frame where the structure is analyzed directly as a general assembly of discrete members.
- **Double layer grids**: A space frame consisting of two planar networks of members forming the top and bottom layers parallel to each other and interconnected by vertical and inclined members.
- **Geodesic dome**: A braced dome in which the elements forming the network are lying on the great circle of a sphere.
- **Lamella**: A unit used to form diamond shaped grids, the size being twice the length of the side of the diamond.
- **Latticed grids**: Double layer grids consisting of intersecting vertical latticed trusses to form regular grids.
- **Latticed shell**: A space frame consisting of curved networks of members built either in single or double layers.
- **Latticed structure**: A structural system in the form of a network of elements whose load-carrying mechanism is three-dimensional in nature.
- **Local buckling**: A snap-through buckling that takes place at one point.
Module: Distance between two joints in the layer of grid.

Overall buckling: Buckling that takes place at a relatively large area where a large number of joints is involved.

Space frame: A structural system in the form of a flat or curved surface assembled of linear elements so arranged that forces are transferred in a three-dimensional manner.

Space grids: Double layer grids consisting of a combination of square or triangular pyramids to form offset or differential grids.

Space truss: A three-dimensional structure assembled of linear elements and assumed as hinged joints in structural analysis.

References


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Further Reading

An introduction to the practical design of space structures is presented in Horizontal-Span Building Structures by W. Schueller. It covers a wide range of topics, including the development, structural behavior, simplified analysis, and application of different types of space structures.

For further study of continuum analogy method of space frames, Analysis and Design of Space Frames by the Continuum Method by L. Kollar and I. Hegedus provides a good reference in this topic.

The quarterly journal International Journal of Space Structures reports advances in the theory and practice of space structures. Special issues treating individual topics of interest were published, such as Stability of Space Structures, V. Gioncu, Ed., 7(4), 1992 and Prefabricated Spatial Frame Systems by A. Hanaor, 10(3), 1995.

Conferences and symposiums are organized annually by the International Association for Shell and Spatial Structures (IASS). The proceedings document the latest developments in this field and provide a wealth of information on theoretical and practical aspects of space structures. The proceedings of conferences recently held are as follows:


Journal of IASS is published three times a year and it covers design, analysis, construction, and other aspects of technology of all types of shell and spatial structures.